

ON THE NONLOCAL PREDICTIONS OF QUANTUM OPTICS

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We give a definition of locality in quantum optics based upon Bell's work, and show that locality has been violated in no experiment performed up to now. We argue that the interpretation of the Wigner function as a probability density gives a very attractive local realistic picture of quantum optics provided that this function is nonnegative. We conjecture that this is the case for all states which can be realized in the laboratory. In particular, we believe that the usual representation of "single photon states" by a Fock state of the Hilbert space is not correct and that a more physical, although less simple mathematically, representation involves density matrices. We study in some detail the experiment showing anticorrelation after a beam splitter and prove that it naturally involves a positive Wigner function. Our (quantum) predictions for this experiment disagree with the ones reported in the literature.

1. What is locality ?

The purpose of this paper is to investigate the conditions for the violation of locality in quantum optics. We shall show that these conditions are rather stringent and have not been fulfilled in those experiments where violations of locality have been claimed.

The first problem is that several, quite different, meanings have been given to the word "locality" (or "nonlocality"). In fact, there are people claiming that quantum mechanics never predicts nonlocality because it forbids sending signals at superluminal velocity. On the other hand, some authors include auxiliary hypotheses, related to the Clauser et al. [1] "no-enhancement" assumption, as a part of the concept of locality. With such a definition there are a lot of locality violations in the predictions of quantum optics. Here we shall use something intermediate between these extremes. We shall define locality in the following form based on Bell's work.

We should consider an EPR (Einstein-Podolsky-Rosen) experiment where some correlation is measured between properties, like spin, of two separated particles. Locality is satisfied if single probabilities (p_3, p_4) and coincidence probabilities (p_{34}) can be obtained from a local hidden variables (LHV) model, i. e., if there are hidden variables, collectively represented by λ , which determine the above probabilities by means of

integrals of the form

$$p_3(\theta_1) = \int W(\lambda) P_3(\lambda, \theta_1) d\lambda, \quad p_4(\theta_2) = \int W(\lambda) P_4(\lambda, \theta_2) d\lambda, \quad (1)$$

$$p_{34}(\theta_1, \theta_2) = \int W(\lambda) P_3(\lambda, \theta_1) P_4(\lambda, \theta_2) d\lambda, \quad (2)$$

the functions P_3 , P_4 and W fulfilling the conditions :

$$\text{Normalization:} \quad \int W(\lambda) d\lambda = 1 \quad (3)$$

$$\text{Positivity:} \quad W(\lambda) \geq 0, \quad P_3(\lambda, \theta_1) \geq 0, \quad P_4(\lambda, \theta_2) \geq 0 \quad (4)$$

$$\text{Boundedness:} \quad P_3(\lambda, \theta_1) \leq 1, \quad P_4(\lambda, \theta_2) \leq 1. \quad (5)$$

2. How to test locality ?

A test of locality involves performing an experiment where quantum optics predicts the violation of some genuine Bell inequality. Genuine means that the inequality can be derived from the conditions (1) to (5) alone, without adding auxiliary assumptions like "no-enhancement" :

$$P_3(\lambda, \theta_1) \leq P_1(\lambda), \quad P_4(\lambda, \theta_2) \leq P_2(\lambda) \quad \text{not assumed.} \quad (6)$$

Here $P_1(\lambda)$ and $P_2(\lambda)$ mean detection probabilities when no selector (e.g. polarizer) is inserted between the source and the detector.

Then, we stress that it is impossible to test locality in experiments measuring coincidences alone. In fact, it has been possible to construct a general LHV model giving the same coincidence probabilities as quantum optics for every EPR-type experiment in which only coincidences are measured [3]. Genuine Bell inequalities, therefore, should necessarily involve both singles and coincidences. In this respect, we do not agree with the usual statement that there are loopholes in the experiments to disprove LHV theories, because the word "loophole" suggests that the experiments have only practical difficulties. The fact is that these experiments have not been designed to test genuine Bell inequalities, but inequalities involving additional assumptions, like (6). Therefore they can only refute restricted families of LHV models, namely those fulfilling those assumptions.

We also point out that a necessary condition [4] for the violation of locality is the existence of an entangled quantum state, i.e., a nonfactorable wavefunction. There are some experiments, where violations of locality have been claimed, which do not even fulfil this condition (e.g. the state vector (2) of Ref. 5 is factorable).

3. Single photon interferometry

The simplest "entangled state" in quantum optics appears in experiments of interference of a single photon (see Fig. 1) [6]. In an experiment of this class, a photon ν_2 is sent to a beam splitter (represented by a dashed line in Fig.1). The state of the radiation field after the beam splitter is

$$|\psi\rangle = 1/\sqrt{2} \{ |0\rangle |1\rangle + |1\rangle |0\rangle \} = 1/\sqrt{2} \{ a_r^\dagger + a_t^\dagger \} |0\rangle |0\rangle, \quad (7)$$

which exhibits entanglement between a "single photon state" and the "vacuum state". In the anticorrelation experiment, represented in Fig. 1, no coincidences are predicted between the detectors PM_r and PM_t , which seems to prove that the photon goes undivided into one channel.

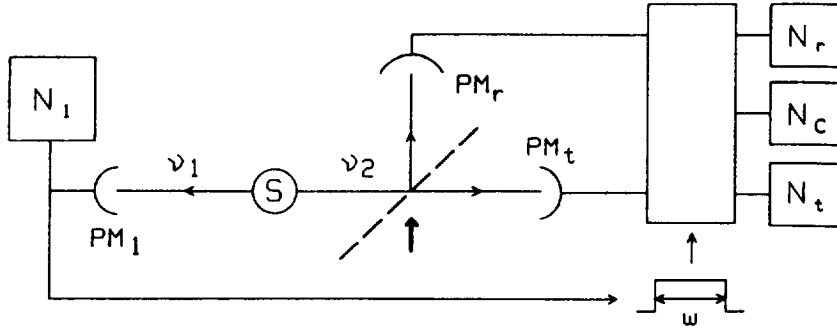


FIG. 1. Triggered experiment [6]. Atoms in the source S produce pairs of photons. The detection of the first photon of the cascade produces a gate during which the photomultipliers, PM_t and PM_r , are active. The probabilities of the detection during the gate are $p_t = N_t/N_1$ and $p_r = N_r/N_1$ for singles and $p_c = N_c/N_1$ for coincidences, N_i being the detection rates.

In the recombination experiment the two beams, produced at the beam splitter, are recombined at another beam splitter where they arrive with a different phase (which may be changed by means of the phase shifter) and the detection probability at one of the outgoing channels depends on that phase difference. The standard way to "explain" this phenomenon is to say that there are two possible routes for the photon and that, according to quantum theory, "possibilities interfere". However, this is not a scientific explanation (at most, it may be considered as a poetic sentence or a practical rule). Later on we shall see how the phenomenon may be *really explained* using the Wigner representation of quantum optics.

4. The Wigner representation as a local hidden variables model

In order to understand the meaning of entanglement in quantum optics, we calculate the Wigner function of (7) and we obtain

$$W = N (2 |\alpha_t + \alpha_r|^2 - 1) \exp(-2 |\alpha_t|^2 - 2 |\alpha_r|^2), \quad N = \text{normalization} \quad (8)$$

This function does not factorize, but this presents no problem; we may interpret W as a joint probability distribution for the amplitudes α_t and α_r . We see that, in the Wigner representation, entanglement is just correlation, an obviously classical concept. Of course, there is another very well known problem, namely that (8) is not positive definite, as a probability should be. We shall return to this difficulty in detail later on, but for the moment let us ignore it proceed as if W were a genuine (non-negative definite)

probability.

In the Wigner representation fields propagate like in classical optics. In fact, the equations involving creation or annihilation operators become, in the Wigner representation, similar equations between field amplitudes:

$$\begin{aligned} a_t^+ &= 1/\sqrt{2} (a_1^+ + i a_0^+) \rightarrow E_t = 1/\sqrt{2} (E_1 + i E_0), \\ a_r^+ &= 1/\sqrt{2} (a_0^+ + i a_1^+) \rightarrow E_r = 1/\sqrt{2} (E_0 + i E_1). \end{aligned} \quad (9)$$

The picture that emerges is that of light as pure (Maxwellian) waves, but with a real zeropoint (background) radiation. Then, in the single photon interference experiment there is always "something" in both channels, because both $\alpha_t \neq 0$ and $\alpha_r \neq 0$.

Anticorrelation can be understood as a result of the interference between signal and zeropoint at the beam splitter. In fact, as shown by Eq. (9), the amplitudes of the fields in the outgoing channels contain a part coming from the signal (channel 1) and a part coming from the zeropoint (channel 0, arrow from below in Fig. 1). The superposition of amplitudes at channels r and t should produce interference, which will depend on the relative phases of the incoming channels. However, in any case the interference will be constructive in one channel and destructive in the other one, by conservation of energy. Now, quantum optics predicts that detectors are only sensitive to the intensity above the zeropoint level, which *explains* why there is detection only in one channel. Detection sensitive only to the intensity above the zeropoint level follows from the normal ordering prescription of quantum optics. In the Wigner representation normal ordering becomes a subtraction of the zeropoint, as shown by the equality

$$a^+a = 1/2(a^+a + a a^+) - 1/2 \rightarrow |\alpha|^2 - 1/2 \quad (10)$$

In this way we have a transparent picture of the anticorrelation, without any need of "photons". The explanation of the interference in the recombination experiment is rather easy, because we are dealing with a purely wave theory of radiation. Interference is produced between (correlated) signal and zeropoint at the second beam splitter. What we want to emphasize is that the only problem of the Wigner function is the lack of positivity, not entanglement. If the Wigner function of an entangled state is nonclassical, it is not because it contains correlation, but because it is not positive definite.

5. The meaning of enhancement and other nonclassical effects

Now it is easy to understand why "no-enhancement" (see Eq. (6)) is violated. In fact from Eq. (7) it follows that

$$I_t = 1/2 [I_1 + I_0 + 2 \operatorname{Re}(E_1 E_0^*)], \quad I_j = |E_j|^2, \quad (11)$$

and this intensity may be greater than the intensity I_1 of the incoming signal, if the relative phase of E_1 and E_0 is zero. A similar phenomenon happens at a polarizer. It is enough to assume that the detection probability increases monotonically with the incoming intensity to explain the origin of

"enhancement".

The existence of a real zeropoint electromagnetic radiation is, therefore, crucial for the explanation of enhancement. On the other hand, all LHV theories in which "no-enhancement" holds true have been refuted by the performed experimental tests of Bell's inequalities [7-9]. Consequently we may conclude that LHV theories not involving zeropoint field are the ones actually refuted by these experiments. If one is fond of LHV theories, one should therefore look for theories involving a real zeropoint. It is very good that the Wigner representation of quantum optics, with the probabilistic interpretation above suggested, belongs to this class.

In the wave interpretation of quantum optics that we are suggesting (taking a positive Wigner function as a probability distribution) the interpretation of the nonclassical states of light is also transparent. According the usual definition, "nonclassical states of light" are those not having a positive Glauber-Sudarshan (P) representation. That is, for nonclassical states, the P-representation either does not exist or it is not non-negative definite. In contrast, any classical state of light has a positive P-representation, $P(\{\alpha_j\})$, which may be interpreted as a probability distribution of the amplitudes of the normal modes of the radiation. If we assume that there is a zeropoint radiation having a probability distribution $W_0(\{\beta_j\})$, in addition to the classical radiation, the question arises: What is the probability distribution of the full radiation present? The answer is obvious, in every normal mode the total amplitude, γ_j , will be the sum of both amplitudes, i.e. $\gamma_j = \alpha_j + \beta_j$. Then the probability distribution of the full radiation will be

$$W(\{\gamma_j\}) = \int P(\{\alpha_j\}) W_0(\{\gamma_j - \alpha_j\}) d^{2N}\alpha . \quad (12)$$

This is just the Wigner function, which is known to be related to the P-function by Eq.(12). Therefore, classical states of light are those where the zeropoint is not modified; some additional radiation is added on top of the zeropoint. Consequently, "nonclassical states" are those where the zeropoint is modified.

6. Solution of the positivity problem

A very simple solution of the positivity problem, the problem that the Wigner function is not positive definite for all quantum states of light, is to assume that only states with a positive Wigner function may be manufactured in the laboratory. That is, we assume that quantum states with a negative Wigner function are just mathematical constructions useful as intermediate steps in some calculations. Two main objections may be put to this assumption, namely that some of these forbidden states have been actually produced, e.g. single photon states, and that there are other representations in quantum optics, e.g. the Q -or positive P- representation, which are always positive and therefore better candidates than the Wigner function. We shall devote the remainder of the paper to answering the first objection. The answer to the second is that the Wigner function has a number of properties that make it the only good candidate. To quote just one, it is the only phase-space distribution which evolves according to a classical Liouville equation for any Hamiltonian quadratic in the creation and

annihilation operators. The Liouville equation corresponds to the classical Hamiltonian obtained from the quantum one by first putting all operators in symmetrical ordering (by using the standard commutation relations) and then replacing the operator a_j (a_j^\dagger) by the classical amplitude α_j (α_j^*). Then, the amplitude α_j^* will be the canonical momentum conjugated to the coordinate α_j .

Now for the first objection. In the first place we point out that we do not propose to interpret the Wigner function throughout quantum mechanics as a probability distribution in phase space. For instance, we do not apply it to the electrons in an atom. We make the proposal just for the electromagnetic field. In contrast with what happens in particle quantum mechanics, in quantum optics most of the states of the radiation have a positive Wigner function. For instance, this is the case for the vacuum, the coherent states, the chaotic state (thermal light) and even the squeezed states. Amongst the usual states of light, practically only Fock states (number states) have a non-positive Wigner function. Then the question arises: can pure Fock states really be produced in the laboratory?. What we conjecture is that Fock states are never produced as pure states, always as mixtures having a positive Wigner function. For instance, if we have a beam of "single-photon signals" such that within a time window w the probability of a signal is $p \ll 1$, then the state corresponding to the window is not the single photon state $|1\rangle$, but the mixture represented by the density matrix $\rho = (1 - p) |0\rangle\langle 0| + p |1\rangle\langle 1|$. The associated Wigner function is positive provided $p < 1/2$. If $p > 1/2$ the probability of having more than one photon within the window becomes relevant and again the Wigner function is positive. It may be argued that, with some effort, it is possible to monitor the single photon signals in such a way that the probability of having one in a time window is close to one whilst the probability of having more than one is negligible. We shall return to this later on.

7. Positivity of the Wigner function in parametric down conversion

There is a general argument showing that the Wigner function may be taken as positive in all experiments involving parametric down conversion. These experiments involve one or several nonlinear crystals where, in quantum language, the process takes place of converting a single photon of frequency ω_0 into two photons of frequencies ω_1 and $\omega_2 = \omega_0 - \omega_1$. The quantum Hamiltonian contains terms with an annihilation operator of the first type of photons and two creation operators. However, in all practical calculations the incoming beam (the pumping) is taken as classical, and the annihilation operator is replaced by a classical amplitude. Consequently, the full Hamiltonian becomes quadratic in the operators of creation and destruction of photons.

Now, it is well known that the Wigner function evolves according to a classical Liouville equation whenever the Hamiltonian is quadratic. On the other hand, the Liouville equation preserves positivity, in the sense that if the Wigner function is positive at a time then it remains positive at any later time. As the initial state (before switching on the pumping) is the vacuum, whose Wigner function is positive, the Wigner function remains positive forever. We should point out that the action of devices like lenses,

mirrors, beam splitters, etc. is linear and, consequently, all of them preserve the positivity of the Wigner function.

It is possible to argue that, strictly speaking, the Hamiltonian associated with the nonlinear crystal is cubic rather than quadratic, and a cubic Hamiltonian does not guarantee the positivity of the Wigner function. This is true, but then the "nonclassical" effects due to negative values of the Wigner function should be relevant in those experiments where a clear disagreement is obtained with the (approximate) quantum predictions obtained using a classical pumping. No experiment of this type has been performed to our knowledge.

8. State of the beam produced in an atomic source

In the following we investigate the positivity of the Wigner function in experiments involving photon beams produced by an atomic source. As a typical example we consider the experiment by Grangier et al. [6], represented in Fig. 1 where the authors claimed to have produced single photon signals, which seems to imply negative Wigner functions.

For simplicity we consider atoms with two states: $|g\rangle$ (ground) and $|e\rangle$ (excited). If at time $t=0$ we have $|e\rangle |0\rangle$ (excited atom plus radiation vacuum), then the evolution gives (to first order perturbation theory)

$$|\psi(t)\rangle = N \{ |e\rangle + |g\rangle A^+(t) \} |0\rangle ; \quad A^+(t) \equiv \sum_j c_j(t) \exp[-ik_j \cdot x] a_j^\dagger \quad (13)$$

where N is a normalization constant, j labels the radiation mode and $A^+(t)$ is the creation operator of a (localized, multimode) photon .

If we consider many atoms, which arrive at the source, are excited there (by the action of a laser) and decay at times $t_1, t_2, \dots, t_s, \dots$, we should represent the atomic beam by the state vector

$$|\psi\rangle = N \prod_s \{ |e_s\rangle + |g_s\rangle A^+(t-t_s) \} |0\rangle \quad (14)$$

If the state of the outgoing atoms are not controlled, (14) is not the correct representation of the physical situation. In fact, we must take the partial trace of $\rho = |\psi\rangle \langle \psi|$ over the atomic states, which leads to

$$\rho = N \{ |0\rangle \langle 0| + \sum_s [A^+(t-t_s) |0\rangle \langle 0| A(t-t_s)] + \sum_s \sum_r [A^+(t-t_s) A^+(t-t_r) |0\rangle \langle 0| A(t-t_r) A(t-t_s)] + \dots \} \quad (15)$$

Furthermore, if the emission times are not controlled, we must average over the times t_1, t_2, \dots

After some algebra [11], we obtain that the final density matrix is

$$\rho_{\text{chaotic}} = \prod_j \{ \sum_{n_j} \bar{n}_j^{n_j} (\bar{n}_j + 1)^{-n_j-1} |n_j\rangle \langle n_j| \} \quad (16)$$

which represents chaotic light, no matter how weak is the beam. The average photon number in mode j , \bar{n}_j , is related to the coefficient c_j . Essential for

the result (16) is to take into account the interference between "photons" coming from different atoms.

It is interesting that the Wigner function of (16) is positive definite, a well known property of chaotic light.

The quantum predictions for the correlation and recombination experiments can be easily obtained. The recombination experiment shows interference with 100% visibility, which is not strange because the state (16) corresponds to "classical" light that can be treated by standard wave optics. The correlation experiment gives for the ratio of the coincidence probability to the product of singles:

$$\alpha \equiv P_{\text{coinc.}} (p_r p_t)^{-1} = 2 \quad \text{if detection window} \ll \text{lifetime of excited atom}$$

$$\alpha \equiv P_{\text{coinc.}} (p_r p_t)^{-1} = 1 \quad \text{if detection window} \gg \text{lifetime of excited atom}$$

For intermediate situations we get values of α between 0 and 1. This result can be also explained by classical optics, as is well known since the early work of Brown-Twiss [10], who showed experimentally the photon bunching properties (i.e., $\alpha > 1$) of chaotic light.

9. "Single photon signals" in an atomic beam

The procedure used by Grangier et al. [6] in order to manufacture single photon signals was to monitor the photons by detecting them in coincidence with partner photon emitted by the atom in a cascade. Of course, our two state model for the atom is no longer adequate because a cascade implies at least three atomic states. However, it is still appropriate to represent the state of the beam by Eq.(15) provided that, in addition to taking the partial trace of the density matrix over the atomic states, we average over all emission times except one, say t_0 .

We get a "single photon signal" superimposed to the chaotic light, which may be represented by the density matrix

$$\rho_{\text{single photon}} = N \{ \rho_{\text{chaotic}} + A(t-t_0) \rho_{\text{chaotic}} A(t-t_0) \} \quad (17)$$

The interesting result is that the Wigner function of (17) is positive.

A straightforward but lengthy calculation[11] gives the quantum prediction for the commented experiments. With the parameters of the actual experiment [6] (that is a coincidence window about twice the atomic lifetime) we predict that, when the beam is very intense, the "single photon" effect is lost and we get the asymptotic value $\alpha \rightarrow 1.57$ (pure chaotic light). In the performed experiment[6] values $\alpha > 1$ are not observed because, in the actual experimental conditions, the spacial coherence in the detector is lost[12]. However, we think that rather modest improvements of the apparatus will allow observing our prediction of interference between photons emitted by different atoms[11].

10. Discussion

We have shown that the interpretation of the Wigner function as a probability distribution provides a very attractive local realistic view of quantum optics. The main difficulty for this interpretation is the fact that the Wigner function is not positive definite for some quantum states. We conjecture that all states actually realizable in the laboratory have a positive Wigner function. In particular we have shown that this is the case for two typical situations where it is claimed that "single photon states" are produced, namely parametric down conversion and light beams produced by a (weak) atomic source. In the second case we argue that the correct representation of the light beam is by means of a density matrix, rather than a pure quantum state. We do not accept the so-called ignorance interpretation of the density matrix, that is as a probability distribution on the set of pure quantum states. On the contrary, we assume that most of the pure quantum states are not physical states.

Even if the above conjecture is correct, and the Wigner function of all physical states of light is nonnegative, some problems remain. We do not understand yet the processes of emission and detection of light or, more generally, the interaction of light with atoms. That is, we do not have a local realistic theory of atoms. In particular, we do not claim to interpret the Wigner function of the electrons in the atom as a probability distribution.

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